$$\frac{\partial^2 v_{e1y}}{\partial t^2} + \Omega_e \frac{\partial v_{e1z}}{\partial t} - \Omega_e \frac{\partial v_{e1z}}{\partial t} + \Omega_e^2 v_{e1y}$$

$$= -\frac{e}{m_e}\frac{\partial E_{1y}}{\partial t} + \frac{e}{m_e}\Omega_e E_{1z}$$

$$\Rightarrow \left[\frac{\partial^2}{\partial t^2} + \Omega_e^2\right] v_{e1y} = -\frac{e}{m_e}\left[\frac{\partial E_{1y}}{\partial t} - \Omega_e E_{1z}\right] \quad (18)$$

again differentiating (17) w.r.t t

we get —

$$\left(\frac{\partial^2}{\partial t^2} + \Omega_e^2\right)\frac{\partial}{\partial t} v_{e1z} = -\frac{e}{m_e}\left[\Omega_e \frac{\partial E_{1y}}{\partial t} + \frac{\partial^2}{\partial t^2} E_{1z}\right]$$

using (14) we get —

$$\left[\frac{\partial^2}{\partial t^2} + \Omega_e^2\right]\left[\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right] E_{1z}$$

$$= \frac{4\pi e^2 n_0}{m_e c^2}\left[\frac{\partial^2 E_{1z}}{\partial t^2} + \Omega_e \frac{\partial E_{1y}}{\partial t}\right]$$

$$= \frac{\omega_p^2}{c^2}\left[\frac{\partial^2 E_{1z}}{\partial t^2} + \Omega_e \frac{\partial E_{1y}}{\partial t}\right] \quad (19)$$

again differentiating (18) w.r.t t

and using (13) we get —

$$\left[\frac{\partial^2}{\partial t^2} + \Omega_e^2\right]\left[\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right] E_{1y} = \frac{\omega_p^2}{c^2}\left[\frac{\partial^2 E_{1y}}{\partial t^2} - \Omega_e \frac{\partial E_{1z}}{\partial t}\right] \quad (20)$$

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using the plasmawave solutions in eqn (19) & (20) we get —

$$\left[-\omega^2+\Omega_e^2\right]\left[-k^2+\frac{\omega^2}{c^2}\right]E_{1z}$$

$$=\frac{\omega_p^2}{c^2}\left[-\omega^2 E_{1x} - \Omega_e i\omega E_{1y}\right] \quad (21)$$

$$\left[-\omega^2+\Omega_e^2\right]\left[-k^2+\frac{\omega^2}{c^2}\right]E_{1y}$$

$$=\frac{\omega_p^2}{c^2}\left[-\omega^2 E_{1y} + \Omega_e i\omega E_{1z}\right] \quad (22)$$

elleminating E_{1x} & E_{1y} from eqn (21) & (22) we get —

$$\begin{vmatrix} \left(-\omega^2+\Omega_e^2\right)\left(-k^2+\frac{\omega^2}{c^2}\right)+\frac{\omega_p^2}{c^2}\omega^2 & \frac{\omega_p^2}{c^2}\Omega_e i\omega \\ +\frac{\omega_p^2}{c^2}\Omega_e i\omega & -\left(-\omega^2+\Omega_e^2\right)\left(-k^2+\frac{\omega^2}{c^2}\right)+\frac{\omega_p^2}{c^2}\omega^2 \end{vmatrix}$$

$$\Rightarrow \left\{\left(-\omega^2+\Omega_e^2\right)\left(-k^2+\frac{\omega^2}{c^2}\right)+\frac{\omega_p^2}{c^2}\omega^2\right\}^2 = 0$$

$$-\frac{\omega^2\Omega_e^2}{c^2}\omega_p^4 = 0$$

$$\Rightarrow \left[\left(-\omega^2+\Omega_e^2\right)\left(\omega^2-c^2k^2\right)+\omega_p^2\omega^2\right]^2$$

$$-\omega_p^4\omega^2\Omega_e^2 = 0$$

$$\Rightarrow \left(\omega^2-k^2c^2\right)\left(\Omega_e^2-\omega^2\right)+\omega_p^2\omega^2$$

$$= \pm\,\omega_p^2\omega\Omega_e$$

$$\Rightarrow (\omega^2 - k^2c^2)(\Omega e^2 - \omega^2) = \pm \omega p^2 \omega \Omega e - \omega^2 \omega p^2$$

Taking +ve sign we get —

$$\omega^2 - k^2c^2 = \frac{\omega p^2 \omega [\Omega e - \omega)}{\Omega e^2 - \omega^2}$$

$$= \frac{\omega p^2}{\frac{\Omega e + \omega}{\omega}} = \frac{\omega p^2}{1 + \frac{\Omega e}{\omega}}$$

Taking -ve sign we get —

$$\omega^2 - k^2c^2 = \frac{\omega p^2}{1 - \frac{\Omega e}{\omega}}$$

Combining the above two eqns we can write —

$$\omega^2 - k^2c^2 = \frac{\omega p^2}{1 \pm \frac{\Omega e}{\omega}} \quad \text{—— (23)}$$

The ± sign indicates that there are two possible solutions of the eqns (19) & (20) corresponding two to different waves that can propagates along $\vec{B}o$

The dispersion relation (23) also can be written as —

$$\tilde{n}^2 = \frac{c^2k^2}{\omega^2} = 1 - \frac{\frac{\omega p^2}{\omega^2}}{1 - \frac{\Omega e}{\omega}} \rightarrow \text{R-wave}$$

$$\tilde{n}^2 = \frac{c^2 k^2}{\omega^2} = 1 - \frac{\frac{\omega_p^2}{\omega^2}}{1 + \frac{\omega_c}{\omega}} \rightarrow \text{L-wave}$$

R-wave means R.H. circular polarisation
L- " " L.H " wave

-: Kinetic theory of plasma :-

Introduction :-

A plasma is a system containing large number of interacting charged particles. The fluid theory provides a simple description of plasma. though

it can be describe the necessity of observe phenomena in plasma by use of fluid theory but it is found that it is inadequate in some cases. For more accurate description of plasma we use statistical kinetic approach. In this approach we have to use the concept of distribution function. (The differential equation satisfied by the distribution function is generally known as Boltzmann equation. (Using kinetic theory of plasma we introduce the collisionless Boltzmann equation is known as Vlasov equation.) As an example of its use we shall derive the dispersion relation for electron plasma waves in a warm plasma and discuss the phenomenon of Landau damping.

Distribution function & Macroscopic variables :-

The one particle distribution function $f(\vec{r},\vec{v},t)$ is so defined that the number of particles within the volume element $d^3r\, d^3v$ of phase space about the phase space co-ordinates $(\vec{r},\vec{v})$ at time t is given by —

$$dn(\vec{r},\vec{v},t) = f(\vec{r},\vec{v},t)\, d^3r\, d^3v \quad \text{——} (1)$$

$$d^3r\, d^3v = dx\, dy\, dz\, dv_x\, dv_y\, dv_z$$

In a statistical sense the distribution function provides a complete description of the system which is under consideration. (The Macroscopic property of the system is due to the average behaviour of the large number of particles whereas the motion of is individual particle is controled by the usual rule of Mechanics

If we know the distribution func $f(\vec{r},\vec{v},t)$ we can able to deduce all the macroscopic variables of physicals interest for the system.

Therefore, the average value of a physical quantity $g(\vec{r},\vec{v},t)$ will be defined by

$$\langle g \rangle = \frac{1}{n}\int g\, f\, d^3v \quad \text{——} (2)$$

where $n = n(\vec{r},t)$ is the number of particles per unit volume about $\vec{r}$ at time t having any velocity.

therefore, $n(\vec{r},t) = \int_v f(\vec{r},\vec{v},t)\, d^3v$

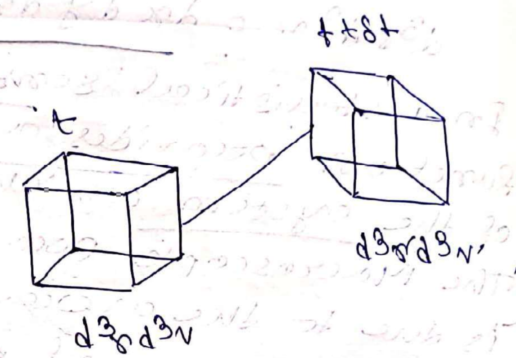
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\vec{r},\vec{v},t)\, dv_x\, dv_y\, dv_z \quad \text{--- (3)}$$

for example,

the average flow velocity

$$u(\vec{r},t) = \frac{1}{n}\int \vec{v}\, f(\vec{r},\vec{v},t)\, d^3v$$

Boltzmann equation :-



In order to describe a system from statistical kinetic approach the distribution function for this system must be known. (The dependence of the distribution function $f(\vec{r},\vec{v},t)$ on $\vec{r}$, $\vec{v}$ and t is governed by an equation known as Boltzmann equation.) The distribution function $f(\vec{r},\vec{v},t)$ is so defined that $f(\vec{r},\vec{v},t)\, d^3r\, d^3v$ gives the number of particles in the volume element $d^3r\, d^3v$ of phase space about the phase space co-ordinates $(\vec{r},\vec{v})$ at time t. Suppose that each particle is subjected to an external force $\vec{F}$. In the absence of collisions with co-ordinates about $(\vec{r},\vec{v})$ in phase space will be found after time interval δt about new co-ordinates $(\vec{r}', \vec{v}')$ such that

$$\vec{r}' = \vec{r} + \vec{v}\,\delta t \quad \text{——— (1)}$$

$$\vec{v}' = \vec{v} + a\,\delta t \quad \text{——— (2)}$$

$\vec{a} = \dfrac{\vec{F}}{m}$ is the field-induced acceleration of the particle of mass m.

In the absence of collisions particles which were in the phase volume $d^3r\, d^3v$ at time t will now occupy the phase volume $d^3r'\, d^3v'$ at time $(t+\delta t)$

therefore,

$$f(\vec{r}', \vec{v}', t+\delta t)\, d^3r'\, d^3v' = f(\vec{r}, \vec{v}, t)\, d^3r\, d^3v$$

for the transformations ——— (3)
eqn (1) & (2) we have defined by

$$d^3r'\, d^3v' = d^3r\, d^3v \quad \text{——— (4)}$$

(according to the 1st order correction of δt)

Thus from eqn (3) we get –

$$f(\vec{r}', \vec{v}', t+\delta t) = f(\vec{r}, \vec{v}, t)$$

$$\Rightarrow f(\vec{r}+\vec{v}\,\delta t, \vec{v}+\vec{a}\,\delta t, t+\delta t) = f(\vec{r}, \vec{v}, t) \quad \text{——— (5)}$$

using (1) & (2)

expanding L.H.S of (5) in Taylor series and keeping the terms of 1st order δt only, we get —

$$f(\vec{r}, \vec{v}, t+\delta t) + \vec{v}\cdot\vec{\nabla} f\,\delta t + \vec{a}\cdot\vec{\nabla}_v f\,\delta t = f(\vec{r}, \vec{v}, t)$$

Where the operator $\vec{\nabla}$ corresponds to differentiations w.r.t x, y & z